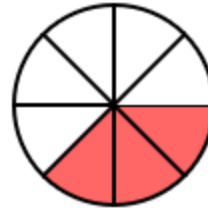
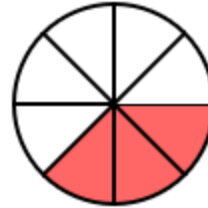


# How to Multiply Fractions

Introducing:

- factor
- product
- reciprocal
- inverse
- identity



$$\frac{3}{8} \times 3$$

**first factor**  
(number in each row)

**second factor**  
(number of rows)

# Multiply Fractions 1



$$\frac{3}{8} \times 3$$

**first factor**                      **second factor**  
(number in each row)      (number of rows)

The parts of this multiplication example are the first *factor*  $\frac{3}{8}$ , and a second *factor* 3. There are 3 rows with  $\frac{3}{8}$  in each row.

# Multiply Fractions 2



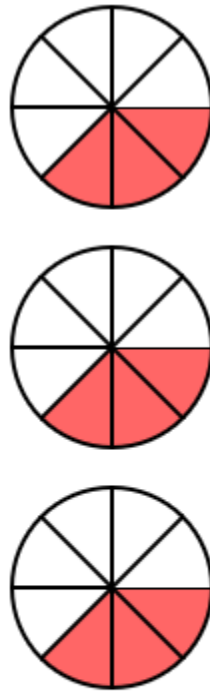
$$\frac{3}{8} \times 3 = 1 \frac{1}{8}$$

first factor      second factor  
(number in each row)      (number of rows)

Multiplication is a form of addition. This picture shows that  $\frac{3}{8}$  is added 3 times. The *product* can be found by addition of like amounts:

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{9}{8}$$

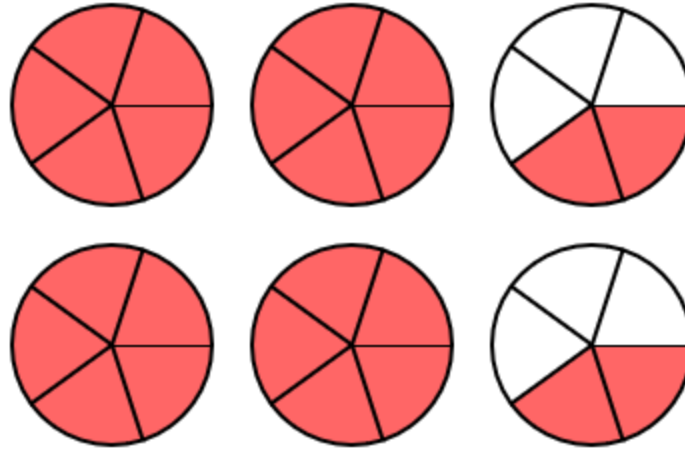
# Multiply Fractions 3



$$\begin{array}{ccccccc} \frac{3}{8} & \times & 3 & = & \frac{3}{8} & \times & \frac{3}{1} = \frac{9}{8} = 1 \frac{1}{8} \\ \text{first factor} & & \text{second factor} & & \text{Write in fraction form and multiply.} & & \text{Simplify.} \\ \text{(number in each row)} & & \text{(number of rows)} & & & & \end{array}$$

To calculate the *product*, write both *factors* in fraction form. Then multiply the numerators 3 and 3 for 9 in the *product* numerator and the denominators 8 and 1 for 8 in the *product* denominator.

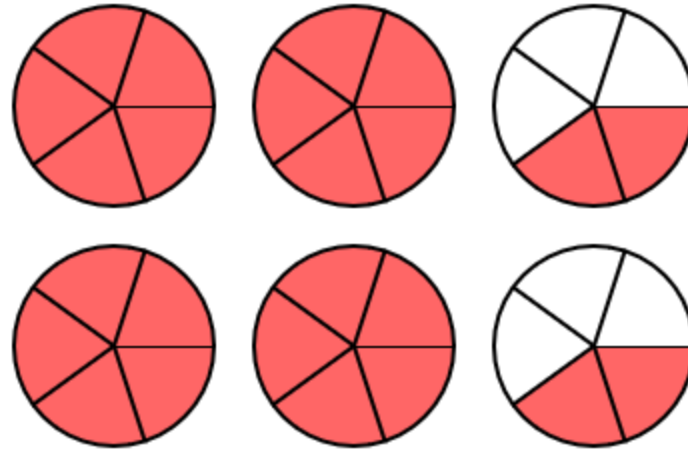
# Multiply Fractions 4



$$2 \frac{2}{5} \times 2 = 4 \frac{4}{5}$$

It is easy to tell the *product*  $4 \frac{4}{5}$  from this picture. Notice the 4 complete circles and the  $\frac{2}{5} + \frac{2}{5}$  circles for a *product* of  $4 \frac{4}{5}$ .

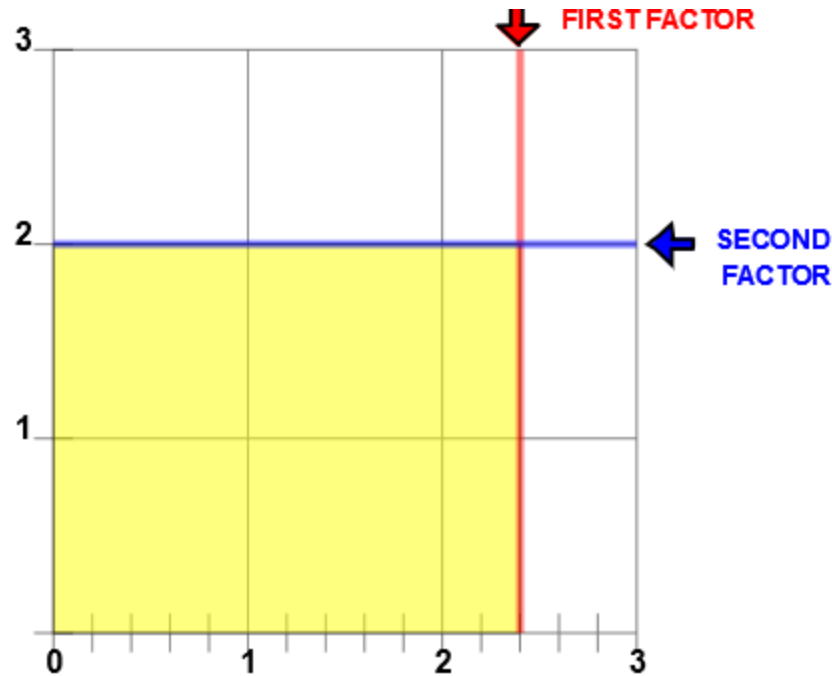
# Multiply Fractions 5



$$\begin{array}{ccccccc}
 2 & \frac{2}{5} & \times & 2 & = & \frac{12}{5} & \times & \frac{2}{1} & = & \frac{24}{5} & = & 4 & \frac{4}{5} \\
 \text{first factor} & & & \text{second factor} & & & & & & & & & \\
 \text{(number in each row)} & & & \text{(number of rows)} & & & & & & & & & \text{Simplify.} \\
 \text{Write in fraction form and multiply.} & & & & & & & & & & & & 
 \end{array}$$

To calculate the *product*, write both *factors* in fraction form. Then multiply the numerators 12 and 2 for 24 in the *product* numerator and the denominators 5 and 1 for 5 in the *product* denominator.

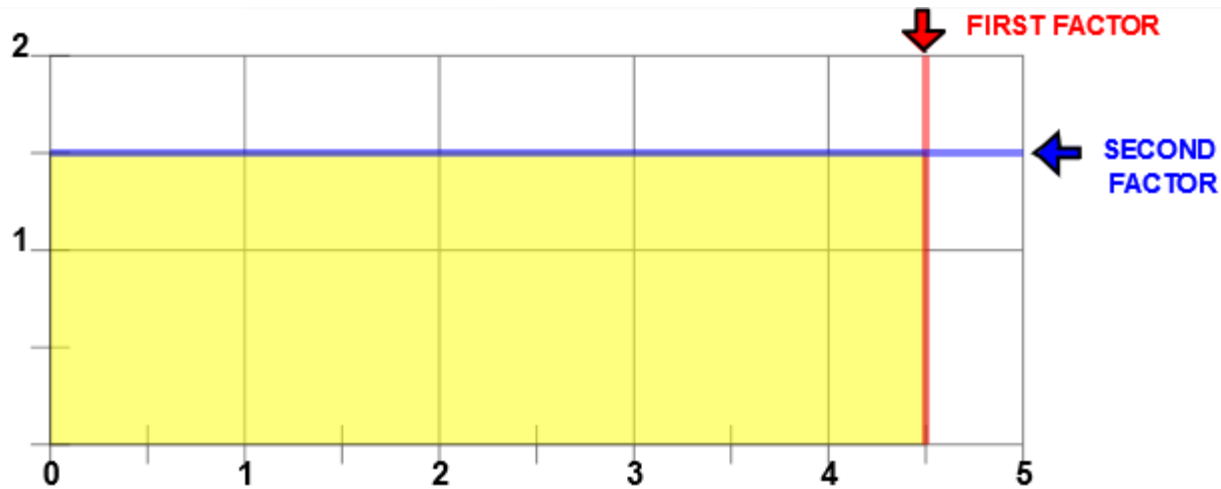
# Multiply Fractions 6



$$\begin{array}{ccccccc}
 2 \frac{2}{5} & \times & 2 \frac{0}{1} & = & \frac{12}{5} & \times & \frac{2}{1} = \frac{24}{5} = 4 \frac{4}{5} \\
 \text{first factor} & & \text{second factor} & & \text{Write in fraction form and multiply.} & & \text{Simplify.} \\
 \text{(horizontal distance)} & & \text{(vertical distance)} & & & & 
 \end{array}$$

The same example,  $2 \frac{2}{5} \times 2$  using a rectangular array. The first *factor*  $2 \frac{2}{5}$ , is shown by the red arrow - the horizontal distance. The second *factor* 2, is shown by the blue arrow - the vertical distance from the bottom. The *product*,  $4 \frac{4}{5}$  is enclosed by the yellow rectangle.

# Multiply Fractions 7



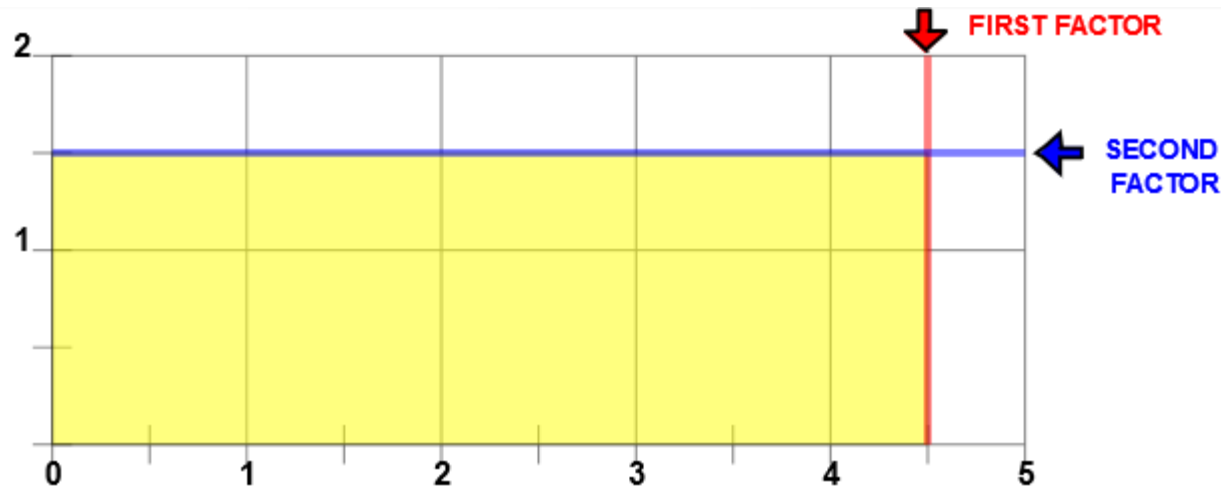
$$4 \frac{1}{2} \times 1 \frac{1}{2} = \frac{9}{2} \times \frac{3}{2} = \frac{27}{4} = 6 \frac{3}{4}$$

**first factor** (horizontal distance)      **second factor** (vertical distance)      **Write in fraction form and multiply.**      **Simplify.**

This rectangular array shows the product of  $4 \frac{1}{2}$  and  $1 \frac{1}{2}$ . Notice how each factor has been written in fraction form before multiplying. You can see in the picture that there are 27 fourths.



# Multiply Fractions 8

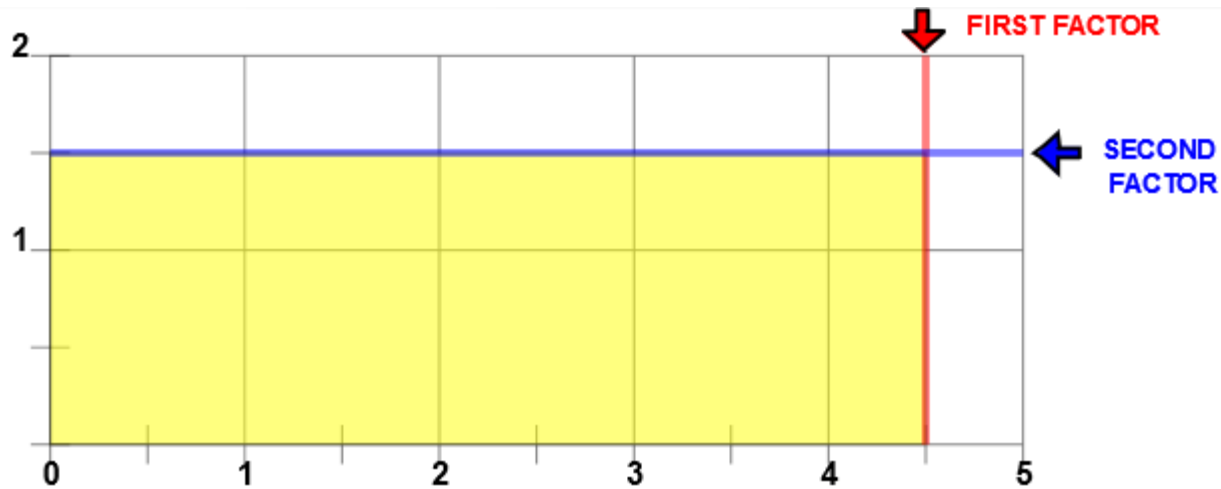


$$4 \frac{1}{2} \times 1 \frac{1}{2} = \frac{9}{2} \times \frac{3}{2} = \frac{27}{4} = 6 \frac{3}{4}$$

**first factor** (horizontal distance)      **second factor** (vertical distance)      **Write in fraction form and multiply.**      **Simplify.**

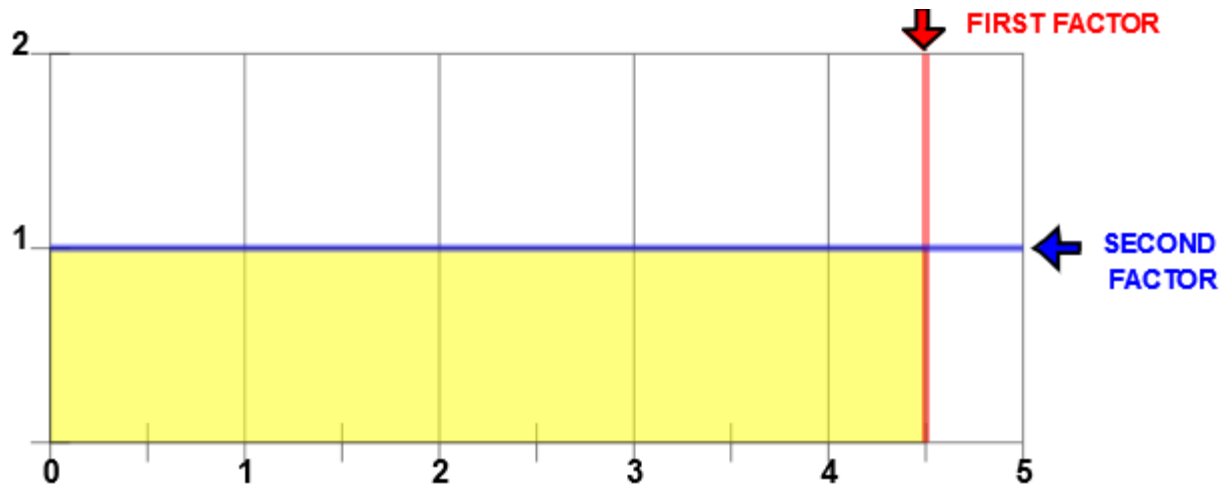
Both *factors* are greater than 1. The *product* is greater than  $4 \times 1$  but less than  $5 \times 2$  by rounding down and rounding up both *factors*. So the product  $6 \frac{3}{4}$  makes sense.

# Multiply Fractions 9



You can tell by the picture from the previous example that there are 4 whole units, five  $\frac{1}{2}$  units, and one  $\frac{1}{4}$  units. The sum of the units is  $4 + \frac{5}{2} + \frac{1}{4} = 6 \frac{3}{4}$ .

# Multiply Fractions 10

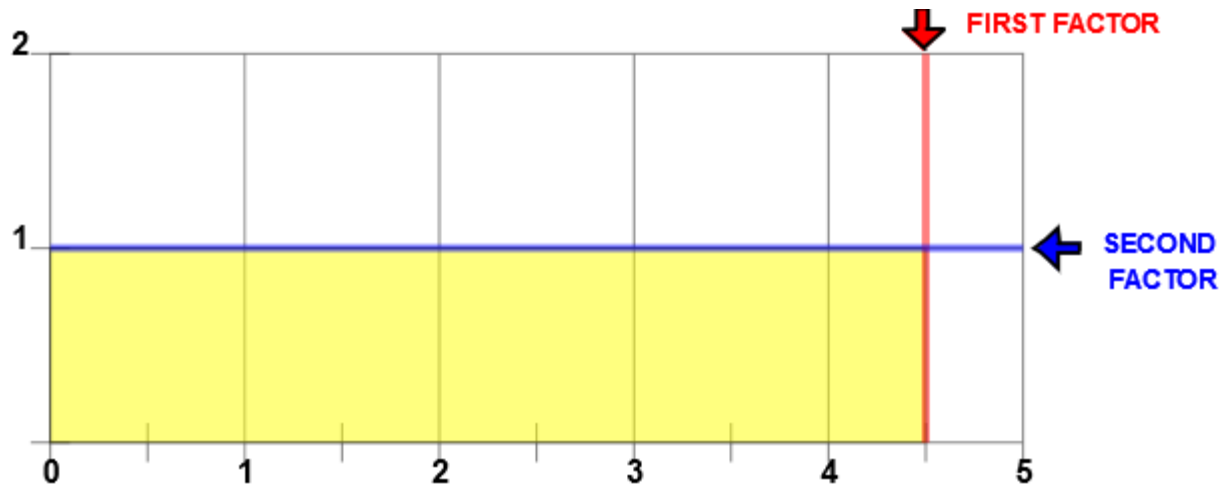


$$4 \frac{1}{2} \times 1 \frac{0}{1} = 4 \frac{1}{2}$$

**first factor** (horizontal distance)      **second factor** (vertical distance)      Multiplying by 1

The second *factor* has been decreased to 1. The *product* has been decreased to  $4 \frac{1}{2}$ .

# Multiply Fractions 11

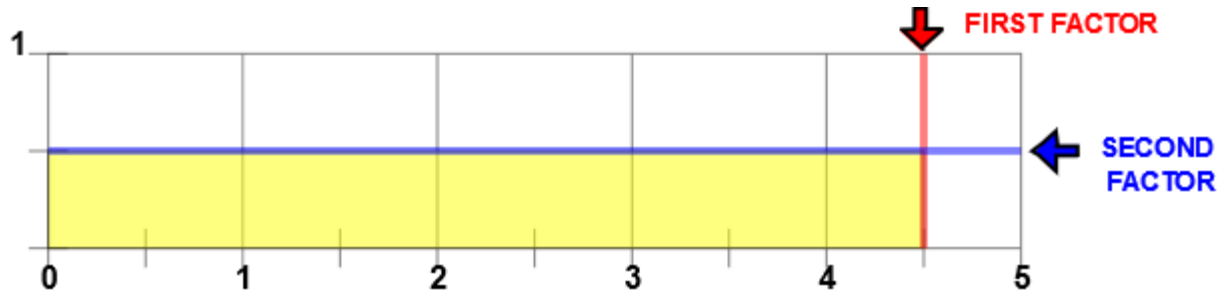


$$4 \frac{1}{2} \times 1 \frac{0}{1} = 4 \frac{1}{2}$$

**first factor**  
**(horizontal distance)**      **second factor**  
**(vertical distance)**      **Multiplying by 1**

When 1 is used as a *factor*, the *product* is equal to the other *factor*. One is called the *identity* for multiplication.

# Multiply Fractions 12

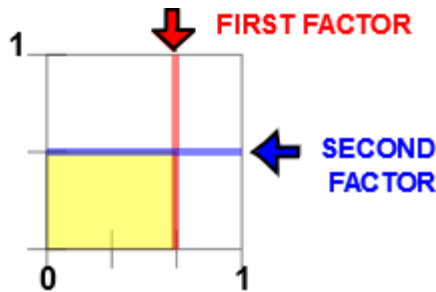


$$4 \frac{1}{2} \times 1 \frac{1}{2} = \frac{9}{2} \times \frac{1}{2} = \frac{9}{4} = 2 \frac{1}{4}$$

**first factor** (horizontal distance)      **second factor** (vertical distance)      Write in fraction form and multiply.      Simplify.

The second *factor* has been decreased to  $\frac{1}{2}$ . Notice the *product* has been decreased to  $2 \frac{1}{4}$ . When one of the *factors* is smaller than 1, the *product* is smaller than the other *factor*.

# Multiply Fractions 13

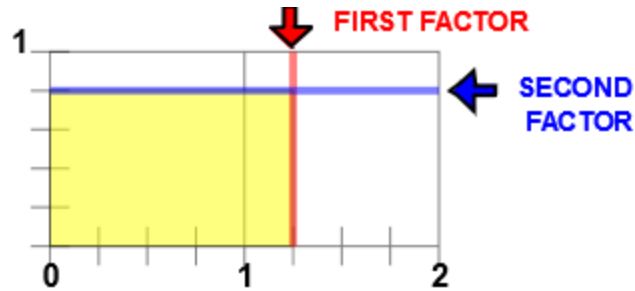


$$0 \frac{2}{3} \times 0 \frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{3} = \frac{1}{3}$$

**first factor** (horizontal distance)      **second factor** (vertical distance)      Write in fraction form, cancel, and multiply. Simplify.

Both *factors* are less than 1. The *product*  $\frac{1}{3}$  is smaller than either *factor*. Notice that the 2 in  $\frac{1}{3}$  and the 2 in  $\frac{1}{3}$  are canceled. See the canceling demonstration in this web site at [visualfractions.com/cancel/](http://visualfractions.com/cancel/)

# Multiply Fractions 14

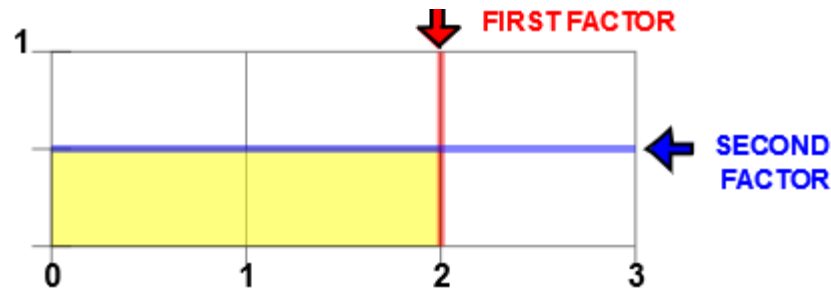


$$1 \frac{1}{4} \times 0 \frac{4}{5} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{1}{1} = 1$$

first factor  
(horizontal distance)
second factor  
(vertical distance)
Write in fraction form, cancel, and multiply. Simplify.

The *factors*  $1 \frac{1}{4}$  and  $\frac{4}{5}$  are *reciprocals*. As you can see, multiplying  $\frac{5}{4}$  by  $\frac{4}{5}$  gives a *product* of 1. If you are asked to *invert* or write the *reciprocal* of  $\frac{5}{4}$  you will write  $\frac{4}{5}$ .

# Multiply Fractions 15

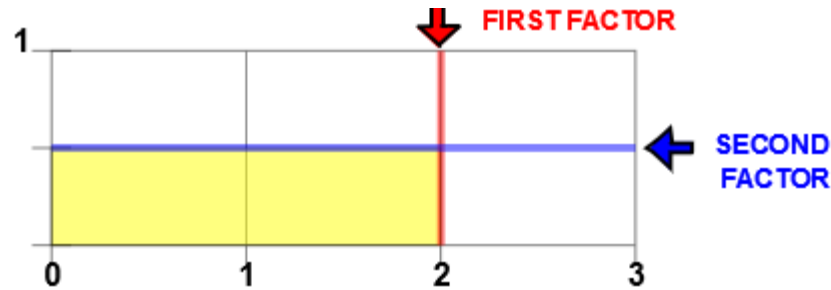


$$\begin{array}{ccccccc}
 2 & \frac{0}{1} & \times & 0 & \frac{1}{2} & = & \frac{1}{1} & \times & \frac{1}{1} & = & \frac{1}{1} & = & 1 \\
 \text{first factor} & & & \text{second factor} & & & & & & & & & & \\
 \text{(horizontal distance)} & & & \text{(vertical distance)} & & & & & & & & & & \\
 & & & & & & & & & & & & & \text{Write in fraction form, cancel, and multiply. Simplify.}
 \end{array}$$

To find the *reciprocal* of a fraction, replace the denominator with the numerator and the numerator with the denominator. The *reciprocal* or *inverse* of  $\frac{2}{1}$  is  $\frac{1}{2}$ .



# Multiply Fractions 16



$$\begin{array}{ccccccc}
 2 \frac{0}{1} & \times & 0 \frac{1}{2} & = & \frac{1}{1} & \times & \frac{1}{2} & = & \frac{1}{1} & = & 1 \\
 \text{first factor} & & \text{second factor} & & & & & & & & \\
 \text{(horizontal distance)} & & \text{(vertical distance)} & & & & & & & & \\
 & & & & \text{Write in fraction form, cancel, and multiply.} & & \text{Simplify.} & & & & 
 \end{array}$$

The picture shows 3 square units. Two  $\frac{1}{2}$  squares are selected. This gives a first *factor* of 2 and a second *factor* of  $\frac{1}{2}$ . Added together,  $\frac{1}{2}$  and  $\frac{1}{2}$  squares give a *sum* of 1 unit.

The *factors* 2 and  $\frac{1}{2}$  are reciprocals because their *product* is 1.

# Multiply Fractions 17

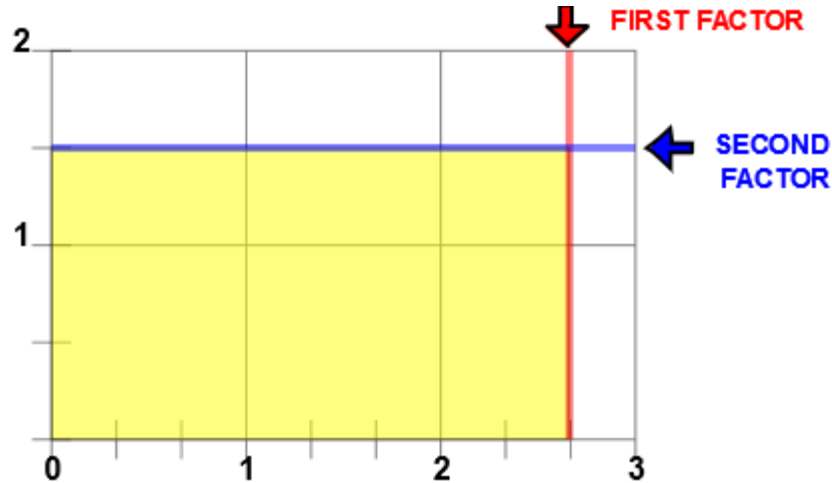
$$2 \frac{2}{3} \times 1 \frac{1}{2} = ?$$

**first factor**  
**(horizontal distance)**

**second factor**  
**(vertical distance)**

What is the product of  $2 \frac{2}{3}$  and  $1 \frac{1}{2}$  ?

# Multiply Fractions 18



$$2 \frac{2}{3} \times 1 \frac{1}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{4}{1} = 4$$

**first factor** (horizontal distance)    **second factor** (vertical distance)    Write in fraction form, cancel, and multiply. Simplify