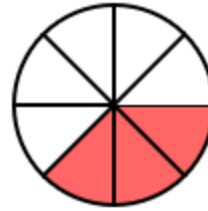
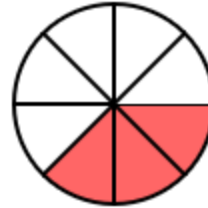


How to Multiply Fractions

Introducing:

- factor
- product
- reciprocal
- inverse
- identity



$$\frac{3}{8} \times 3$$

first factor
(number in each row)

second factor
(number of rows)

Multiply Fractions 1



$$\frac{3}{8} \times 3$$

first factor **second factor**
(number in each row) **(number of rows)**

The parts of this multiplication example are the first *factor* $\frac{3}{8}$, and a second *factor* 3. There are 3 rows with $\frac{3}{8}$ in each row.

Multiply Fractions 2



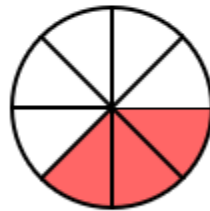
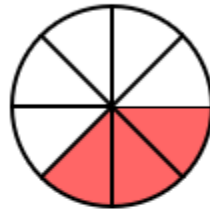
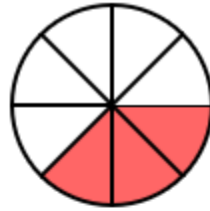
$$\frac{3}{8} \times 3 = 1 \frac{1}{8}$$

first factor second factor
(number in each row) (number of rows)

Multiplication is a form of addition. This picture shows that $\frac{3}{8}$ is added 3 times. The *product* can be found by addition of like amounts:

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{9}{8}$$

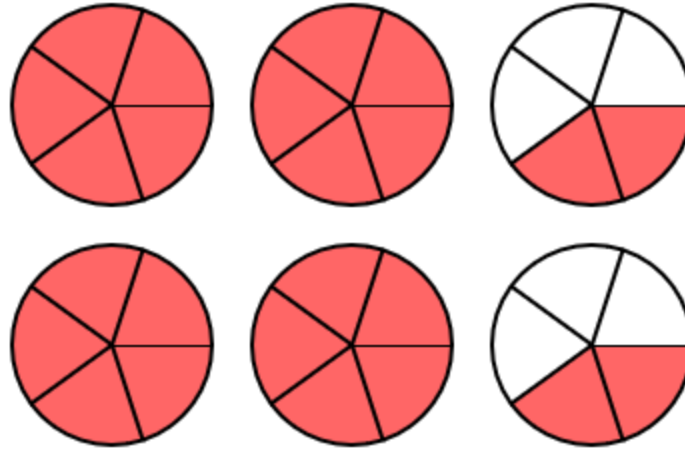
Multiply Fractions 3



$$\begin{array}{ccccccc} \frac{3}{8} & \times & 3 & = & \frac{3}{8} & \times & \frac{3}{1} = \frac{9}{8} = 1 \frac{1}{8} \\ \text{first factor} & & \text{second factor} & & \text{Write in fraction form and multiply.} & & \text{Simplify.} \\ \text{(number in each row)} & & \text{(number of rows)} & & & & \end{array}$$

To calculate the *product*, write both *factors* in fraction form. Then multiply the numerators 3 and 3 for 9 in the *product* numerator and the denominators 8 and 1 for 8 in the *product* denominator.

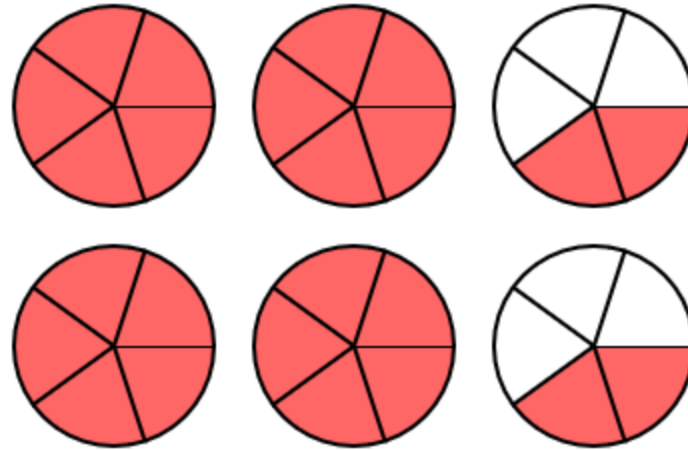
Multiply Fractions 4



$$2 \frac{2}{5} \times 2 = 4 \frac{4}{5}$$

It is easy to tell the *product* $4 \frac{4}{5}$ from this picture. Notice the 4 complete circles and the $\frac{2}{5} + \frac{2}{5}$ circles for a *product* of $4 \frac{4}{5}$.

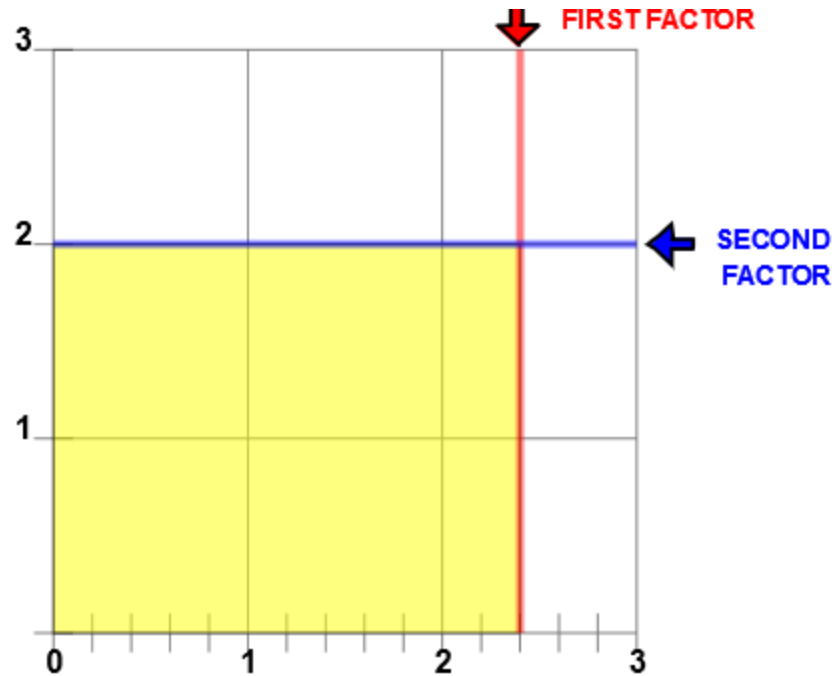
Multiply Fractions 5



$$\begin{array}{ccccccc}
 2 & \frac{2}{5} & \times & 2 & = & \frac{12}{5} & \times & \frac{2}{1} & = & \frac{24}{5} & = & 4 & \frac{4}{5} \\
 \text{first factor} & & & \text{second factor} & & & & & & & & & \\
 \text{(number in each row)} & & & \text{(number of rows)} & & & & & & & & & \text{Simplify.} \\
 \text{Write in fraction form and multiply.} & & & & & & & & & & & &
 \end{array}$$

To calculate the *product*, write both *factors* in fraction form. Then multiply the numerators 12 and 2 for 24 in the *product* numerator and the denominators 5 and 1 for 5 in the *product* denominator.

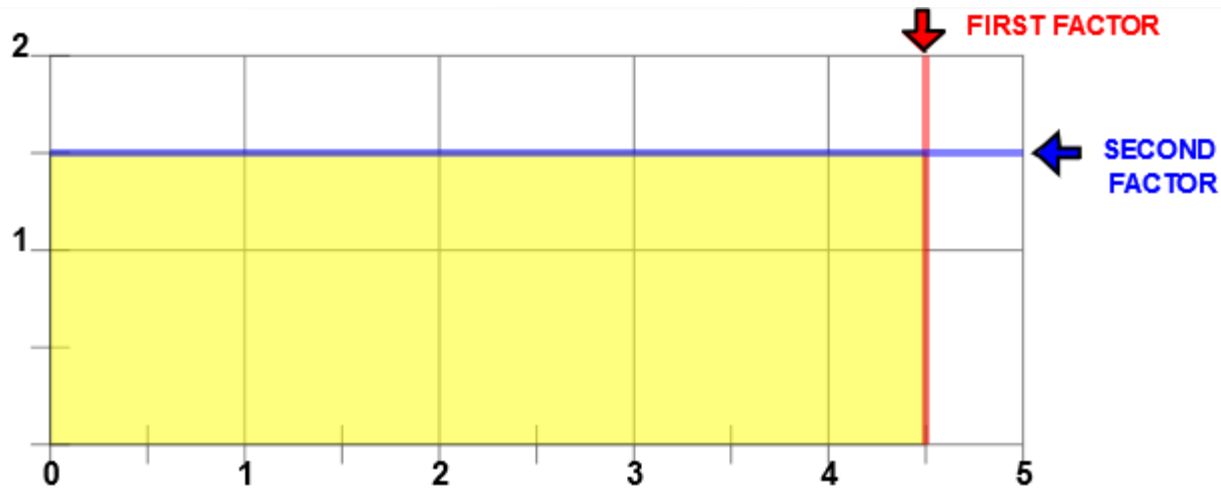
Multiply Fractions 6



$$\begin{array}{ccccccc}
 2 \frac{2}{5} & \times & 2 \frac{0}{1} & = & \frac{12}{5} & \times & \frac{2}{1} = \frac{24}{5} = 4 \frac{4}{5} \\
 \text{first factor} & & \text{second factor} & & \text{Write in fraction form and multiply.} & & \text{Simplify.} \\
 \text{(horizontal distance)} & & \text{(vertical distance)} & & & &
 \end{array}$$

The same example, $2 \frac{2}{5} \times 2$ using a rectangular array. The first *factor* $2 \frac{2}{5}$, is shown by the red arrow - the horizontal distance. The second *factor* 2, is shown by the blue arrow - the vertical distance from the bottom. The *product*, $4 \frac{4}{5}$ is enclosed by the yellow rectangle.

Multiply Fractions 7

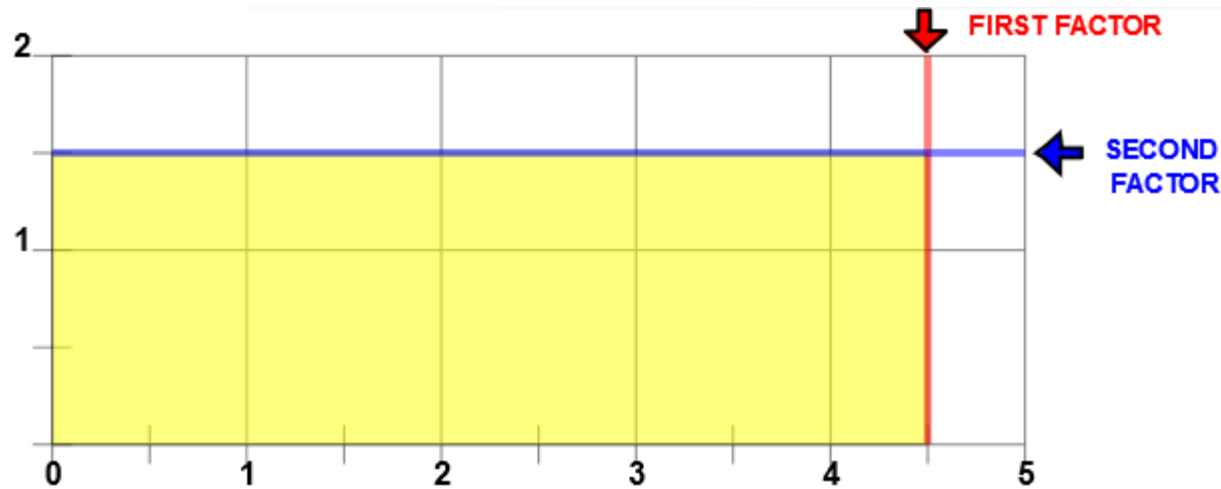


$$4 \frac{1}{2} \times 1 \frac{1}{2} = \frac{9}{2} \times \frac{3}{2} = \frac{27}{4} = 6 \frac{3}{4}$$

first factor (horizontal distance) **second factor** (vertical distance) **Write in fraction form and multiply.** **Simplify.**

This rectangular array shows the product of $4 \frac{1}{2}$ and $1 \frac{1}{2}$. Notice how each factor has been written in fraction form before multiplying. You can see in the picture that there are 27 fourths.

Multiply Fractions 8

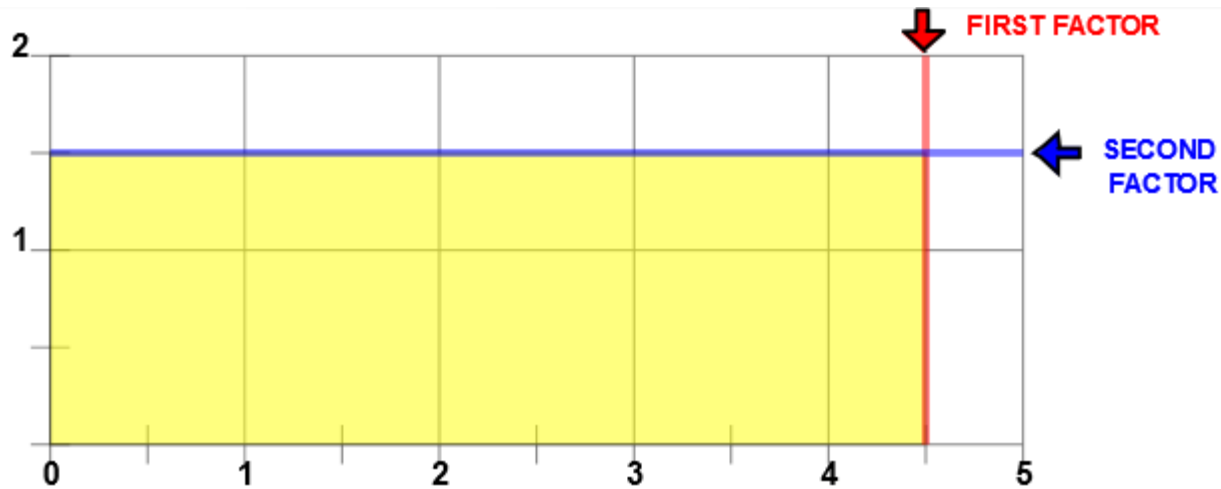


$$4 \frac{1}{2} \times 1 \frac{1}{2} = \frac{9}{2} \times \frac{3}{2} = \frac{27}{4} = 6 \frac{3}{4}$$

first factor (horizontal distance) **second factor** (vertical distance) **Write in fraction form and multiply.** **Simplify.**

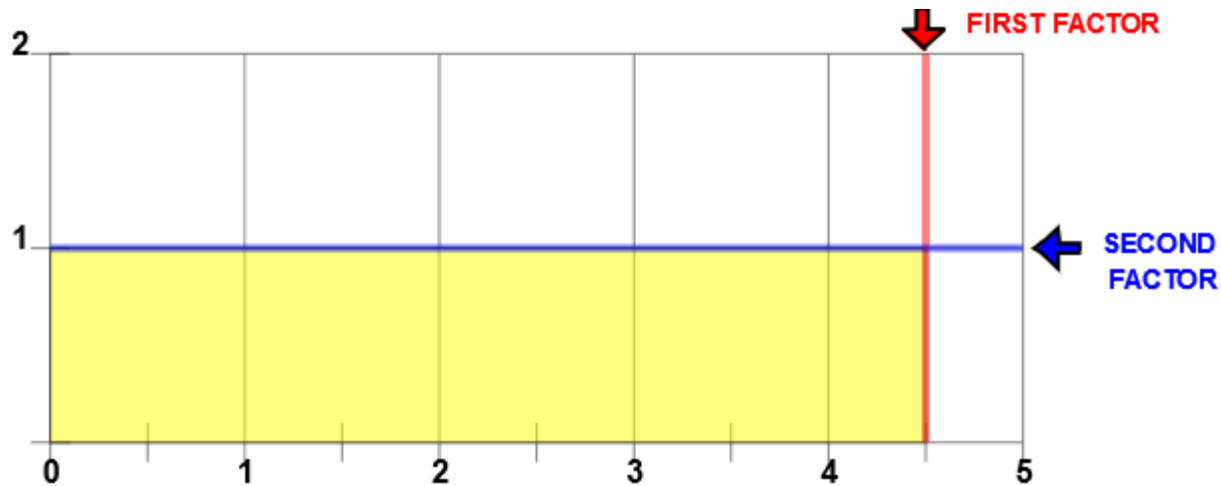
Both *factors* are greater than 1. The *product* is greater than 4×1 but less than 5×2 by rounding down and rounding up both *factors*. So the product $6 \frac{3}{4}$ makes sense.

Multiply Fractions 9



You can tell by the picture that there are 4 whole units, five $\frac{1}{2}$ units, and one $\frac{1}{4}$ units. The sum of the units is $4 + \frac{5}{2} + \frac{1}{4} = 6 \frac{3}{4}$.

Multiply Fractions 10

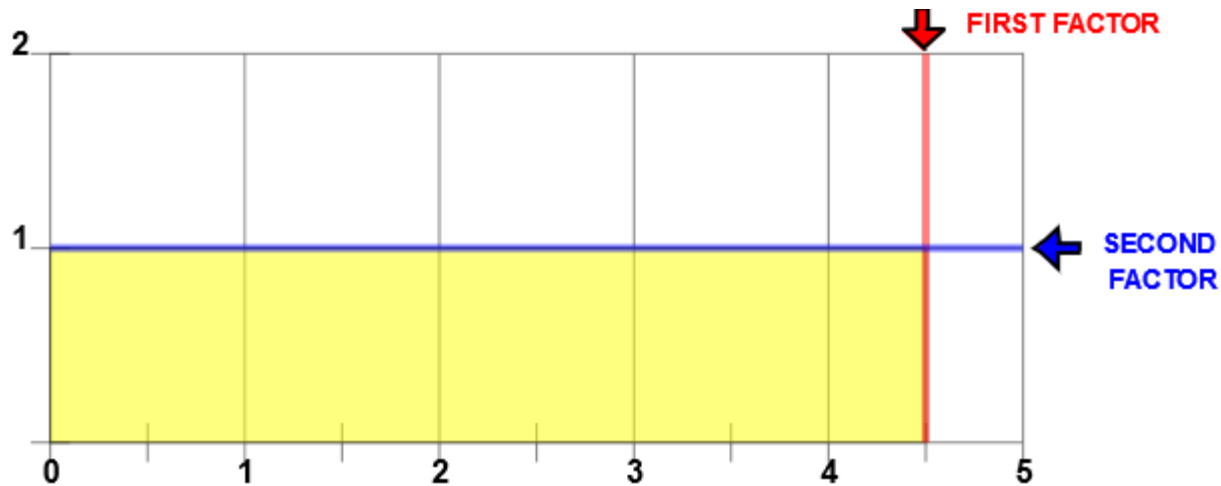


$$4 \frac{1}{2} \times 1 \frac{0}{1} = 4 \frac{1}{2}$$

first factor (horizontal distance) **second factor** (vertical distance) Multiplying by 1

The second *factor* has been decreased to 1. The *product* has been decreased to $4 \frac{1}{2}$.

Multiply Fractions 11

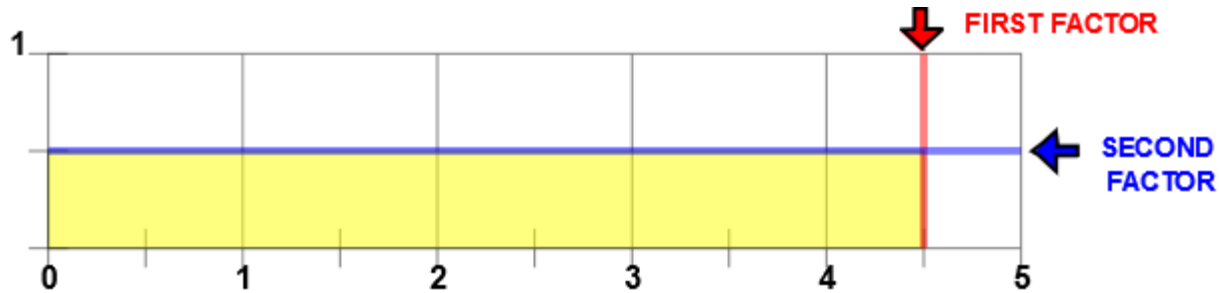


$$4 \frac{1}{2} \times 1 \frac{0}{1} = 4 \frac{1}{2}$$

first factor (horizontal distance) **second factor** (vertical distance) Multiplying by 1

When 1 is used as a *factor*, the *product* is equal to the other *factor*. One is called the *identity* for multiplication.

Multiply Fractions 12

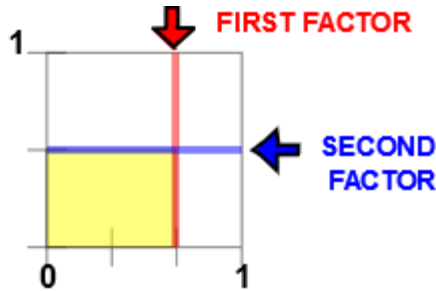


$$4 \frac{1}{2} \times 1 \frac{1}{2} = \frac{9}{2} \times \frac{1}{2} = \frac{9}{4} = 2 \frac{1}{4}$$

first factor (horizontal distance) **second factor** (vertical distance) **Write in fraction form and multiply.** **Simplify.**

The second *factor* has been decreased to $\frac{1}{2}$. Notice the *product* has been decreased to $2 \frac{1}{4}$. When one of the *factors* is smaller than 1, the *product* is smaller than the other *factor*.

Multiply Fractions 13

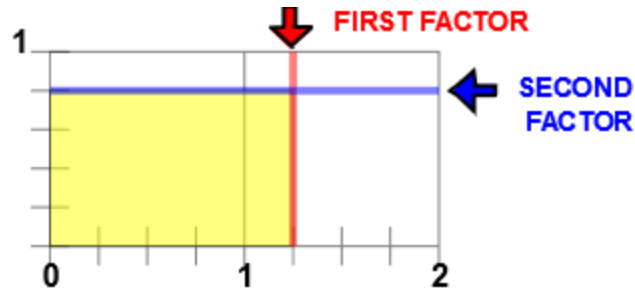


$$\begin{array}{ccccccc} 0 & \frac{2}{3} & \times & 0 & \frac{1}{2} & = & \frac{1}{3} \\ \text{first factor} & & & \text{second factor} & & & \\ \text{(horizontal distance)} & & & \text{(vertical distance)} & & & \end{array} = \frac{\overset{1}{\cancel{2}}}{3} \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{1}{3} = \frac{1}{3}$$

Write in fraction form, cancel, and multiply. Simplify.

Both *factors* are less than 1. The *product* $\frac{1}{3}$ is smaller than either *factor*. Notice that the 2 in $\frac{1}{3}$ and the 2 in $\frac{1}{3}$ are canceled. See the canceling demonstration in this web site at visualfractions.com/cancel/

Multiply Fractions 14

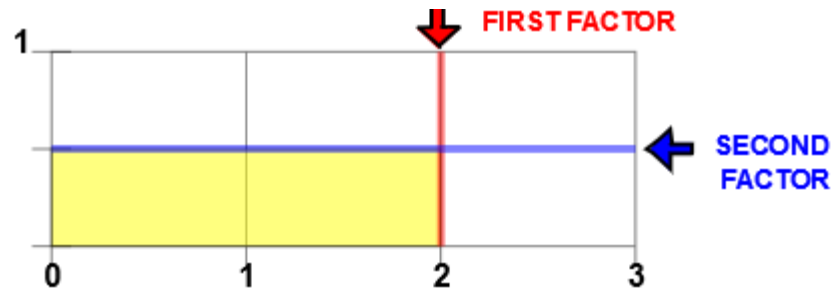


$$1 \frac{1}{4} \times 0 \frac{4}{5} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{1}{1} = 1$$

first factor
(horizontal distance)
second factor
(vertical distance)
Write in fraction form, cancel, and multiply. Simplify.

The *factors* $1 \frac{1}{4}$ and $\frac{4}{5}$ are *reciprocals*. As you can see, multiplying $\frac{5}{4}$ by $\frac{4}{5}$ gives a *product* of 1. If you are asked to *invert* or write the *reciprocal* of $\frac{5}{4}$ you will write $\frac{4}{5}$.

Multiply Fractions 15

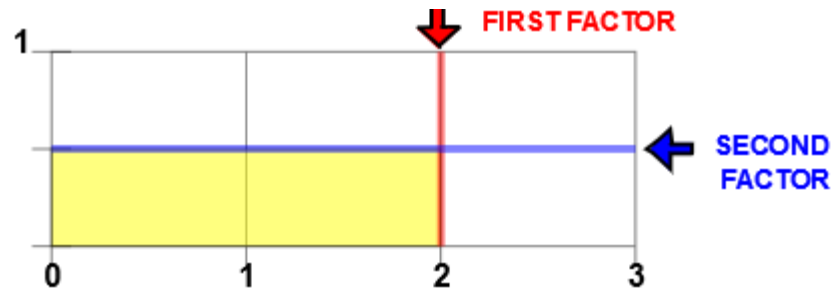


$$2 \frac{0}{1} \times 0 \frac{1}{2} = \frac{1}{1} \times \frac{1}{1} = \frac{1}{1} = 1$$

first factor (horizontal distance) **second factor** (vertical distance) Write in fraction form, cancel, and multiply. Simplify.

To find the *reciprocal* of a fraction, replace the denominator with the numerator and the numerator with the denominator. The *reciprocal* or *inverse* of $2/1$ is $1/2$.

Multiply Fractions 16



$$\begin{array}{ccccccc}
 2 \frac{0}{1} & \times & 0 \frac{1}{2} & = & \frac{1}{1} & \times & \frac{1}{2} & = & \frac{1}{1} & = & 1 \\
 \text{first factor} & & \text{second factor} & & & & & & & & \\
 \text{(horizontal distance)} & & \text{(vertical distance)} & & & & & & & & \\
 & & & & \text{Write in fraction form, cancel, and multiply.} & & & & & & \text{Simplify.}
 \end{array}$$

The picture shows 3 square units. Two $\frac{1}{2}$ squares are selected. This gives a first *factor* of 2 and a second *factor* of $\frac{1}{2}$. Added together, $\frac{1}{2}$ and $\frac{1}{2}$ squares give a *sum* of 1 unit.

The *factors* 2 and $\frac{1}{2}$ are reciprocals because their *product* is 1.

Multiply Fractions 17

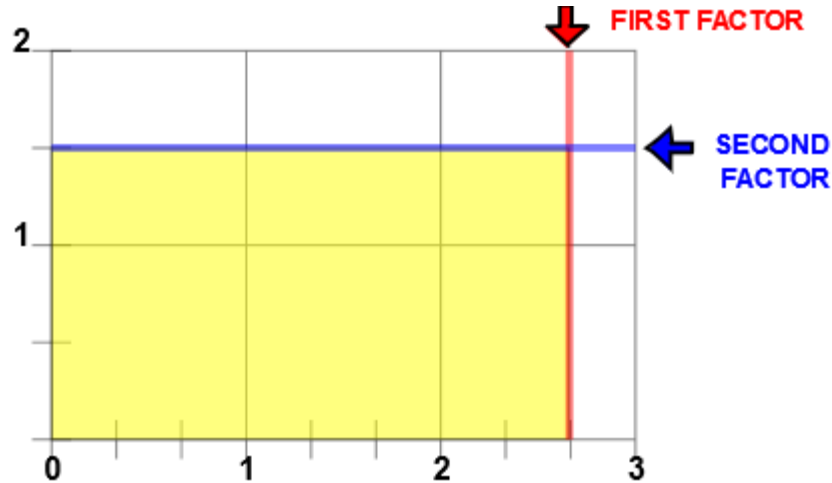
$$2 \frac{2}{3} \times 1 \frac{1}{2} = ?$$

first factor
(horizontal distance)

second factor
(vertical distance)

What is the product of $2 \frac{2}{3}$ and $1 \frac{1}{2}$?

Multiply Fractions 18



$$2 \frac{2}{3} \times 1 \frac{1}{2} = \frac{8}{3} \times \frac{3}{2} = \frac{4}{1} = 4$$

first factor (horizontal distance) **second factor** (vertical distance) Write in fraction form, cancel, and multiply. Simplify